III. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Let the equation to tangent DE be $(x/a)\cos\phi+(y/b)\sin\phi=1$.
Then $CB=a\sec\phi$; $CD=b\operatorname{cosec}\phi$. $DG=b+b\operatorname{cosec}\phi$.
Area$=DG.GE$. $GE : CB :: DG : DC$. Whence

$$GE=\frac{a\sec\phi(1+\operatorname{cosec}\phi)}{\operatorname{cosec}\phi}=\frac{a\sec\phi(1+\operatorname{cosec}\phi)}{\operatorname{cosec}\phi}b(1+\operatorname{cosec}\phi)=ab\tan\phi(1+\operatorname{cosec}\phi)^2$$

Hence $\tan\phi(1+\operatorname{cosec}\phi)^2$ is to be examined for a minimum. Differentiating and equating to zero, we have

$$\sec^2\phi(1+\operatorname{cosec}\phi)^2-2\tan\phi(1+\operatorname{cosec}\phi)\operatorname{cosec}\phi\cot\phi=0,$$

whence, $\sec^2\phi(1+\operatorname{cosec}\phi)=2\operatorname{cosec}\phi$, or

$$\frac{1+\sin\phi}{\sec\phi\cos^2\phi}-\frac{2}{\sin\phi}.$$

Solving, $\sin\phi=\frac{1}{2}$; whence $\operatorname{cosec}\phi=2$.
Hence $DG=b+b\operatorname{cosec}\phi=b+2b=3b$.

Also solved by *P. S. BERG*, *W. H. DRANE*, *J. SCHEFFER*, and *J. W. YOUNG*.

95. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

A ship starts at the equator and sails northeast at all times. How far has the ship sailed (in miles) when her latitude is 30°, 45°, 60°, 90°? How far when her longitude is 90°, 180°, 270°, 360°? Regarding the earth as a sphere, radius 3956 miles.

Solution by the PROPOSER.

Let A be the point of the ship's departure, $APQR$ the ship's course, P, Q two consecutive points on the course, $AG=\theta=$longitude of P, $PG=\phi=$latitude of P, $OP=r=$radius of the earth, $\angle PQN=\beta=\frac{1}{4}\pi$.

Then $PQ=ds$, $PN=EP\times\angle PEN=r\cos\phi d\theta$, $QN=rd\phi$.

$\therefore\ ds^2=r^2(\cos^2\phi d\theta^2+d\phi^2)$.

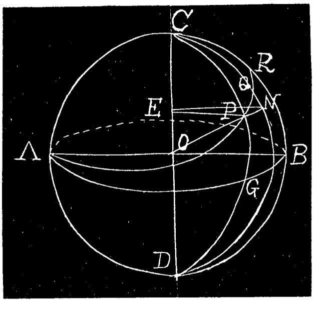
$\therefore\ ds=r(\cos^2\phi d\theta^2+d\phi^2)^{\frac{1}{2}}$ .....(1).

$PN/QN=\tan\beta=\cos\phi d\theta/d\phi$.

$\therefore\ d\theta=\tan\beta d\phi/\cos\phi$......(2).

(2) in (1) gives $ds=\sqrt{(1+\tan^2\beta)}d\phi=rd\phi/\cos\beta$.

$\therefore\ s=(r/\cos\beta)\int_0^\phi d\phi=r\phi/\cos\beta=r\phi\sqrt{2}$.



When $\phi=\frac{1}{6}\pi$, $s=\frac{1}{6}\pi r\sqrt{2}=2929.3817$ miles.
When $\phi=\frac{1}{4}\pi$, $s=\frac{1}{4}\pi r\sqrt{2}=4394.07257$ miles.
When $\phi=\frac{1}{3}\pi$, $s=\frac{1}{3}\pi r\sqrt{2}=5858.7634$ miles.
When $\phi=\frac{1}{2}\pi$, $s=\frac{1}{2}\pi r\sqrt{2}=8788.14514$ miles.

From (2) $d\phi=\cos\phi d\theta/\tan\beta$......(3).
(3) in (1) gives

$$ds=r\cos\phi\sqrt{(1+\tan^2\beta)}d\theta/\tan\beta=r\cos\phi d\theta/\sin\beta.$$

From (2), $\theta=\tan\beta\int\frac{d\phi}{\cos\phi}=\tan\beta\log[\tan(\frac{1}{4}\pi+\frac{1}{2}\phi)]$.

$\therefore\ e^{\theta\cot\beta}=\tan(\frac{1}{4}\pi+\frac{1}{2}\phi)=\frac{1+\sin\phi}{\cos\phi}$. $\therefore\ \cos\phi=\frac{2}{e^{\theta\cot\beta}+e^{-\theta\cot\beta}}=\frac{2}{e^{\theta}+e^{-\theta}}$,

since $\cot\beta=1$.

$$\therefore\ s=2r\sqrt{2}\int_0^{\theta}\frac{d\theta}{e^{\theta}+e^{-\theta}}=2r\sqrt{2}(\tan^{-1}e^{\theta}-\tfrac{1}{4}\pi).$$

When $\theta=\frac{1}{2}\pi$, $s=2r\sqrt{2}(\tan^{-1}e^{\frac{1}{2}\pi}-\frac{1}{4}\pi)=r\sqrt{2}(.3695185\pi)=6494.764423$ miles.
When $\theta=\pi$, $s=2r\sqrt{2}(\tan^{-1}e^{\pi}-\frac{1}{4}\pi)=r\sqrt{2}(.472506\pi)=8304.902620$ miles.
When $\theta=\frac{3}{2}\pi$, $s=2r\sqrt{2}(\tan^{-1}e^{\frac{3}{2}\pi}-\frac{1}{4}\pi)=r\sqrt{2}(.494281\pi)=8687.626341$ miles.
When $\theta=2\pi$, $s=2r\sqrt{2}(\tan^{-1}e^{2\pi}-\frac{1}{4}\pi)=r\sqrt{2}(.498804\pi)=8767.1239018$ miles.

Also solved by *J. SCHEFFER*. A somewhat different solution of this problem is given in Finkel's *Mathematical Solution Book*, page 344.

96. Proposed by **W. H. CARTER**, Vice President, and Professor of Mathemetics, Centenary College, Jackson, La.

If $f(x)=\int f(x)dx$, find $f(x)$, the constant being zero.

Solution by **W. F. SHAW**, 1600 Sabine Street, Austin, Tex.

$f(x)=\int f(x)dx$. $df(x)=f(x)dx$.

$\frac{df(x)}{f(x)}=dx$. $\log f(x)=x$. $f(x)=e^x$.

Also solved by *W. H. DRANE*, *J. SCHEFFER*, and *G. B. M. ZERR*.

MECHANICS.

87. Proposed by **H. C. WHITAKER, M. E., Ph. D.**, Professor of Mathematics, Manual Training School, Philadelphia, Pa.

"He on his impious foes onward drove,
Drove them before him to the bounds
And crystal walls of Heaven; which opening wide
Rolled inward and a spacious gap disclosed
Into the wasteful deep; headlong themselves they threw
Down from the verge of Heaven.
Nine days they fell; Hell at last
Yawning received them whole and on them closed!"
Paradise Lost, Book VI.

Assuming Hell to be the center of the earth, and the only force acting on the lost spirits to be that of gravity due to the earth's attraction,—How far is Heaven ?